CONSTITUTIVE EQUATIONS OF SOLID PROPELLANTS WITH VOLUME DILATATION UNDER MULTIAXIAL LOADINGS - PART 2. DETERMINATION 01'1'111 SHEAR PART OF THE POTENT1/11, FUNCTION IN TERMS OF VOLUME-PRESERVED PRINCIPAL STRETCH RATIOS*

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ABSTRACT

Solid propellants under tension loading dilate significantly. Therefore, they may be aptly called dilatable materials. The amount of volume dilatation depends not only on the extent of deformation, but also on the type of multiaxial loading (i. e., axiality). A volume-prcscrl'cd set of three invariant was previously introduced as a new set of independent variables in the potential function. By using the three volume-preserved invariant, the potential function could be separated into the dilatation part and the shear part. Furthermore, the dilatation part of the potential function was determined and theory of dilation was developed. In this paper, the shear part of the potential function is determined, using a new set of volume-preserved principal stretch ratios.

INTRODUCTION

Solid propellants behave like binonlinear dilatable materials, i.e., they dilate significantly under tension loading; however, they arc almost incompressible under hydrostatic pressure. The dilatation comes mainly from vacuolar formation between binder and particle. Initially, the solid propellant is a highly filled, two-phase composite with binder and particles. However, under tension loading, dewetting occurs and vacuoles are formed. Hence, the propellant becomes a three-phase composite with an additional void phase that results in a large volume increase. Moreover, the amount of volume dilatation depends not only on the extent of deformation, but also on the type of multiaxial loading, i.e., axiality. (It has been observed that volume dilatation under uniaxial tension is significantly lower than that under equal biaxial tension). It behaves binonlinearly at the onset of dewetting. Before the onset of dewetting, it may be assumed to be almost incompressible. After dewetting, it becomes dilatable. It has been shown that a potential function exists which can determine the stress-strain response of solid propellant for different modes of deformation. Since the propellant is dilatable, the dilatation (volume change) shall be included in the development of the potential function.

In the previous paper.['1 a volume-preserved set of three invariants was introduced as a new set of independent variables in the potential function. By using the three volume-preserved invariant, we were able to separate the potential function into the dilatation part and the shear part. Furthermore, the dilatation part of the potential function was determined and the theory of dilatation was developed.[1] Also in the subsequent paper, the shear part of the potential function is determined in terms of invariant.[2] In this paper, the shear part of the potential function is determined in terms of the volume-preserved principal stretch ratios. The purpose of developing the potential function in terms of principal stretch ratios, in part, is due to the two approaches available in the current market of the nonlinear finite element computer codes, i.e., (1) the invariant approach, and (2) the principal stretch ratio approach. Moreover, in this paper, two approaches have been taken to develop the potential function in terms of principal stretch ratios, i.e., (1) the analytical representation ($\Sigma\mu n\lambda_i^{\alpha n}$) and (2), the numerical-graphical representation. Detailed developments are presented here.

In order to present the theory in an orderly fashion, a summary of previous developments is presented in the following sections.

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NONLINEAR THEORY OF DILATABLE MATERIALS

It has been stated that propellants behave like binonlinear dilatable materials, Under critical stress, dewetting occurs, and the propellants change from a two-phase to a three-phase composite with an additional void phase. Since the material is dilatable, the volume-preserved invariants ($\Gamma_1, \Gamma_2, \Gamma_3$) are used for the development Of the theory.

In classical nonlinear elastic theory, the nonlinear stress-strain equation is given by

$$T_{ij} = \frac{2}{I_3^{1/2}}\left[\left(I_2 \frac{\partial W}{\partial I_2} + I_3 \frac{\partial W}{\partial I_3}\right)\delta_{ij} - I_3 \frac{\partial W}{\partial I_3} B_{ij}^{-1} + \frac{\partial W}{\partial I_1} B_{ij}\right] \qquad (1)$$

$$(i, j = 1, 2, 3)$$

where $W = W(I_1, I_2, I_3)$ is potential energy function and the three principal invariant I_1, I_2, I_3 are given by

$$\begin{aligned} I_2 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\ I_2 &= \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 \\ I_3 &= \lambda_1^2 \lambda_2^2 \lambda_3^2 \end{aligned} \qquad (2)$$

where λ_i are the three principal stretch ratios.

Now, the volume-preserved invariants $\Gamma_1, \Gamma_2, \Gamma_3$ are defined by

$$\Gamma_1 = \frac{I_1}{I_3^{1/3}}, \quad \Gamma_2 = \frac{I_2}{I_3^{2/3}}, \quad \Gamma_3 = I_3 \qquad (3)$$

Through the chain rule, the nonlinear stress-strain equation becomes [1]

$$T_{ij} = \frac{2}{\Gamma_3^{1/2}}\left[\left(\frac{\Gamma_2}{3}\frac{\partial W}{\partial \Gamma_2} - \frac{\Gamma_1}{3}\frac{\partial W}{\partial \Gamma_3} + \Gamma_3 \frac{\partial W}{\partial \Gamma_3}\right)\delta_{ij} \Gamma_1^{-1/3} \frac{\partial W}{\partial \Gamma_1} B_{ij} - \Gamma_3^{-1/3} \frac{\partial W}{\partial \Gamma_2} B_{ij}^{-1}\right] \qquad (4)$$

where B_{ij} is a left Cauchy-Green strain tensor.

From the above formulation, one can deduce that the potential function, W, can be given by [1]

$$W = B\, g(\Gamma_1, \Gamma_2) f\left(\frac{\Delta V}{V_0}\right) + H(\Gamma_1, \Gamma_2) \qquad (5)$$

where

$$B\, g(\Gamma_1, \Gamma_2)\, f\left(\frac{\Delta V}{V_0}\right) = \text{dilatation part of the potential function} \qquad (6)$$

$$H(\Gamma_1, \Gamma_2) = \text{shear part of the potential function} \qquad (7)$$

B	=	the bulk modulus
$g(\Gamma_1, \Gamma_2)$	=	an anisotropic function to account for the change in overall hulk modulus due to deformation
$f\left(\frac{\Delta V}{V_0}\right)$	=	a function of volume change $\frac{\Delta V}{V_0}$ - only

Thus, for the complete determination of material, the functions H,f,g, and [he constant B have to be determined.

MODELING APPROACH

It is interesting to observe that by using volume-preserved invariants, 1,, one can express [he nonlinear stress-strain equations under uniform deformation in three distinct types:

Type 1: A general expression which includes the same order-of-magnitude contribution from the dilatation part and shear part of the potential function,

Type II: The trace of stress tensor provides an expression whose sole contributor is the dilatation par[of [he potential function.

Type 111: The difference between two principal stresses provides an expression whose main contributor is the shear part of the potential function.

Because of the unique property of Type 11 equations, the dilatation part of the potential function was determined first (presented in a previous paper).[1] Once the dilatation part of the potential function is determined, one can proceed to use a Type III stress-strain expression to determine the shear part of the potential function because, in a Type III expression, the contribution of the shear part of the potential function is predominate in the stress-strain equation.

Furthermore, in order to assess the internal consistency of the potential energy function, the dilatation and shear part of the potential energy functions (which are obtained independently) are substituted into the general expression of Type I, where this expression provides the same order-of-magnitude contribution from both the dilatation and the shear part of the potential energy function. It is shown that the total potential energy function describes the nonlinear biaxial response quite well.

In the following, the explicit expressions of stress-strain are given for the three types of equations:

Potential energy function: $W = B\,g(\Gamma_1,\Gamma_2)\,f\left(\Gamma_3^{1/2}-1\right)+H(\Gamma_1,\Gamma_2)$

(1) Type I (general expression):

$$
\begin{aligned}
T_{11} &= \frac{2}{\Gamma_3^{1/2}}\left[\left(\frac{\Gamma_2}{3}\frac{\partial W}{\partial \Gamma_2}-\frac{\Gamma_1}{3}\frac{\partial W}{\partial \Gamma_1}+\Gamma_3\frac{\partial W}{\partial \Gamma_3}\right)+\Gamma_3^{-1/3}\frac{\partial W}{\partial \Gamma_1}B_{11}-\Gamma_3^{1/3}\frac{\partial W}{\partial \Gamma_2}B_{11}^{-1}\right] \\
T_{22} &= \frac{2}{\Gamma_3^{1/2}}\left[\left(\frac{\Gamma_2}{3}\frac{\partial W}{\partial \Gamma_2}-\frac{\Gamma_1}{3}\frac{\partial W}{\partial \Gamma_1}+\Gamma_3\frac{\partial W}{\partial \Gamma_3}\right)+\Gamma_3^{-1/3}\frac{\partial W}{\partial \Gamma_1}B_{22}-\Gamma_3^{1/3}\frac{\partial W}{\partial \Gamma_2}B_{22}^{-1}\right] \qquad (8) \\
T_{33} &= \frac{2}{\Gamma_3^{1/2}}\left[\left(\frac{\Gamma_2}{3}\frac{\partial W}{\partial \Gamma_2}-\frac{\Gamma_1}{3}\frac{\partial W}{\partial \Gamma_1}+\Gamma_3\frac{\partial W}{\partial \Gamma_3}\right)+\Gamma_3^{-1/3}\frac{\partial W}{\partial \Gamma_1}B_{33}-\Gamma_3^{1/3}\frac{\partial W}{\partial \Gamma_2}B_{33}^{-1}\right]
\end{aligned}
$$

where B_{ij} is a left Cauchy-Green strain tensor.

(2) Type II (expression from the dilatation part of the potential energy function):

$$\frac{T_{11} + T_{22} + T_{33}}{3} = 2\Gamma_3^{1/2} \frac{\partial W}{\partial \Gamma_3} = B\, g(\Gamma_1, \Gamma_2)\, f'\left(\frac{\Delta V}{V_0}\right) \tag{9}$$

(3) Type III (expression mainly from the shear part of the potential energy function):

$$\begin{aligned}
T_{11} - T_{22} &= \frac{2}{\Gamma_3^{1/2}} \left[\Gamma_3^{-1/3} \frac{\partial W}{\partial \Gamma_1} (B_{11} - B_{22}) - \Gamma_3^{1/3} \frac{\partial W}{\partial \Gamma_2} \left(B_{11}^{-1} - B_{22}^{-1}\right)\right] \\
T_{22} - T_{33} &= \frac{2}{\Gamma_3^{1/2}} \left[\Gamma_3^{-1/3} \frac{\partial W}{\partial \Gamma_1} (B_{22} - B_{33}) - \Gamma_3^{1/3} \frac{\partial W}{\partial \Gamma_2} \left(B_{22}^{-1} - B_{33}^{-1}\right)\right] \\
T_{33} - T_{11} &= \frac{2}{\Gamma_3^{1/2}} \left[\Gamma_3^{-1/3} \frac{\partial W}{\partial \Gamma_1} (B_{33} - B_{11}) - \Gamma_3^{1/3} \frac{\partial W}{\partial \Gamma_2} \left(B_{33}^{-1} - B_{11}^{-1}\right)\right]
\end{aligned} \tag{lo}$$

DILATATION PART OF THE POTENTIAL FUNCTION

In order to determine the volume change due to stress and strain fields, one first determines the dilatation part of the potential function in Eq. (6). It is interesting to note that by taking the trace of Eq. (4), one obtains:

$$\frac{T_{ii}}{3} = B\, g(\Gamma_1, \Gamma_2)\, f'\left(\frac{\Delta V}{V_0}\right), \ (i = 1, 2, 3) \tag{11}$$

which exclusively relates the trace of stress to the dilatation potential function. Therefore, we shall take advantage of the simplified relation to determine the functions of $f\left[\frac{\Delta V}{V_0}\right]$, g (1'], 1'2), and the constant B.

Expressing Eq. (1 1) in terms of principal stresses for various uniform deformations, one has:

3-D: General *Nonequal Triaxial Tension*

$$\frac{T_{11} + T_{22} + T_{33}}{3} = B\, g(\Gamma_1, \Gamma_2)\, f'\left(\frac{\Delta V}{V_0}\right) \tag{12}$$

3-D: *Uniform Triaxial Tension*

$$T = 13\, f'\left(\frac{\Delta V}{V_0}\right), [g(3,3) = 1] \tag{13}$$

2-D: Nonequal *Biaxial Tension*

$$\frac{T_{11} + T_{22}}{3} = B\, g(\Gamma_1, \Gamma_2)\, f'\left[\frac{\Delta V}{V_0}\right) \tag{14}$$

2-D: *Equal Biaxial Tension*

$$\frac{2T_{11}^{EB}}{3} = B\, g(\Gamma_1, \Gamma_2)\, f'\left(\frac{\Delta V}{V_0}\right) \tag{15}$$

In the previous paper,[1] we used the above equations to determine function $f\left(\frac{\Delta V}{V_0}\right)$, $g(I'_1, I'_2)$, and the constant B, which are given by the following explicit forms:

$B = 287.69$ psi,

$$f\left(\frac{\Delta V}{V_0}\right) = \frac{1}{1.5}\left(\frac{\Delta V}{V_0}\right)^{1.5} \tag{16}$$

$$g(I'_1, I'_2) = -2696.9\left(\frac{I'_1}{3}\right) + \left[2155.9\left(\frac{I'_1}{3}\right)^2 + 3183.2\left(\frac{I'_2}{3}\right)\right] + \left[451.1\left(\frac{I'_1}{3}\right)^3 - 3092.36\left(\frac{I'_1}{3}\right)\left(\frac{I'_2}{3}\right)\right] \tag{17}$$

where B is an induced bulk modulus after dewetting, anti was obtained for the very first time for the solid propellants.

In the next section, the shear part of the potential function will be determined using the Type III stress- strain expression.

DETERMINATION OF THE SHEAR PART OF THE POTENTIAL FUNCTION, H, IN TERMS OF THE VOLUME-PRESERVED PRINCIPAL STRETCH RATIOS, λ_i^*

As mentioned previously in the modeling approach, we first determined the dilatation part of the potential function, using a Type 11 stress strain expression. After the functions g and f have been determined, the shear part of the potential function H will be determined. In the previous paper,[2] H was determined in terms of volume-preserved invariant I'_1 and I'_1. Now H will be determined in terms of the stretch ratios. Moreover, the stretch ratio approach can be implemented by two methods: (1) the analytical function representation, and (2) the numerical-graphical representation. In fact, the numerical-graphical approach provides a much more powerful and realistic tool to determine the potential function, using a simple recursion formula.

To model the shear part of the potential energy function, H, in terms of the principal stretch ratios, it is necessary to use the voiume-preserved stretch ratios. $\left(\lambda_i^* = \frac{\lambda_i}{I_3^{1/6}}\right)$ as independent variables, i.e.. $H(\lambda_1^*, \lambda_2^*, \lambda_3^*)$. It is because, from the derivation of the general potential energy function in terms of new invariants I'_i, it is found that H shall be constant under uniform tension or compression. Under these conditions, [he invariants I'_1 and I'_2 are constant and the condition of H as constant is automatically satisfied.

Therefore, if H expressed in terms of volume-preserved stretch ratios, then the constancy of H under uniform tension or compression is automatically satisfied also. Furthermore, to formulate H in terms of λ_i^*, an assumption of separate and symmetric function is proposed, i.e., $H(\lambda_1^*, \lambda_2^*, \lambda_3^*) = h(\lambda_1^*) + h(\lambda_2^*) + h(\lambda_3^*)$. The equation implies that once the function $h(\lambda_i^*)$ is determined, the function $H(\lambda_1^*, \lambda_2^*, \lambda_3^*)$ is aiso determined. As mentioned before, to determine the function $h(\lambda_i^*)$, two approaches are employed. One is to use explicit analytical expression. The other is to use a recursion formula derived from uniaxial and equal biaxial expression which determine the function h (X*) in numerical- graphical form from the uniaxial and equal biaxial data. A detailed mathematical derivation is given for both approaches.

Now, the potential function expressed in terms of the volume-preserved stretch ratios is given by

$$W = Bg\,(I'_1, I'_2)\, f\left(\frac{\Delta V}{V_0}\right) + \left[h(\lambda_1^*) + h(\lambda_2^*) + h(\lambda_3^*)\right] \tag{18}$$

where

$$I'_1 = \frac{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}{I_3^{1/3}} = \lambda_1^{*2} + \lambda_2^{*2} + \lambda_3^{*3} \tag{19}$$

$$I'_2 = \frac{\lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2}{I_3^{2/3}} = \lambda_1^{*2}\lambda_2^{*2} + \lambda_2^{*2}\lambda_3^{*2} + \lambda_3^{*2}\lambda_1^{*2} \tag{20}$$

i. e., the dilatation part of the potential function can be expressed in terms of λ_i^* through I'_1, I'_2, and I'_3. In order to separate the contribution from the dilatation part of the potential function for the stretch ratio expression, wc use the following expression in uniform and homogeneous deformations,

$$T_{11} = \frac{1}{\lambda_2\lambda_3}\frac{\partial W}{\partial \lambda_1} \qquad J\,T_{11} = \lambda_1 \frac{\partial W}{\partial \lambda_1} \tag{21a}$$

$$T_{22} = \frac{1}{\lambda_1\lambda_3}\frac{\partial W}{\partial \lambda_2} \quad \Rightarrow \quad J\,T_{22} = \lambda_2 \frac{\partial W}{\partial \lambda_2} \tag{21b}$$

$$T_{33} = \frac{1}{\lambda_1\lambda_2}\frac{\partial W}{\partial \lambda_3} \qquad J\,T_{33} = \lambda_3 \frac{\partial W}{\partial \lambda_3} \tag{21c}$$

where

$$J = \lambda_1\lambda_2\lambda_3 \tag{21d}$$

Substituting the potential energy function, Eq. (18) into the stress-strain Eq. (21) and subtracting Eq. (21a) from (21 b), or (2 lb) from (2 lc), etc. (which is Type 111 representation), one has

$$JT_{ii} - JT_{jj} = \lambda_i\left\{B\left(\frac{\partial g}{\partial \lambda_i}\right) f\left(\frac{\Delta V}{V_0}\right) + Bg\left(\frac{\partial f}{\partial \lambda_i}\right)\right\} - \lambda_j\left\{B\left(\frac{\partial g}{\partial \lambda_j}\right) f\left(\frac{\Delta V}{V_0}\right) + Bg\left(\frac{\partial f}{\partial \lambda_j}\right)\right\} + \lambda_i\frac{\partial H}{\partial \lambda_i} - \lambda_j\frac{\partial H}{\partial \lambda_j} \tag{22}$$

$(i, j = 1, 2, 3)$

where ii and jj are not summation signs.

For simplicity, one writes

$$J\text{-}l'ii - JT_{jj} \equiv A_{ij} + \lambda_i \frac{\partial H}{\partial \lambda_i} - \lambda_j \frac{\partial H}{\partial \lambda_j} \tag{23}$$

where

$$A_{ij} \equiv \lambda_i \left\{ B\left(\frac{\partial g}{\partial \lambda_i}\right) f\ \left(\frac{\Delta V}{V_0}\right) + Bg\left(\frac{\partial f}{\partial \lambda_i}\right) \right\} - \lambda_j \left\{ B\left(\frac{\partial g}{\partial \lambda_j}\right) f\ + Bg \frac{\partial f}{\partial \lambda_j} \right\} \tag{24}$$

which are the contributions from the dilation part of the potential energy function.

For the function $H(\lambda_i^*)$, one can express through the chain rule

$$\lambda_1 \frac{\partial H(\lambda_m^*)}{\partial \lambda_1} = \sum_{j=1}^{3} \frac{\partial H(\lambda_m^*)}{\partial \lambda_j^*} \frac{\partial \lambda_j^*}{\partial \lambda_1}$$

$$= \frac{2}{3} \lambda_1^* \frac{\partial H}{\partial \lambda_1^*} - \frac{1}{3} \lambda_2^* \frac{\partial H}{\partial \lambda_2^*} - \frac{1}{3} \lambda_3^* \frac{\partial H}{\partial \lambda_3^*} \tag{25}$$

Therefore

$$\lambda_i \frac{\partial H}{\partial \lambda_i} - \lambda_j \frac{\partial H}{\partial \lambda_j} = \lambda_i^* \frac{\partial H}{\partial \lambda_i^*} - \lambda_j^* \frac{\partial H}{\partial \lambda_j^*} \text{ (summation sign does not apply)} \tag{26}$$

Also with a separate, symmetric function $h(\lambda_i^*)$, one has

$$\lambda_i \frac{\partial H}{\partial \lambda_i} - \lambda_j \frac{\partial H}{\partial_j} = \lambda_i^* h'(\lambda_i^*) - \lambda_i^* h'(\lambda_j^*) \tag{27}$$

The next step is to determine the contribution from the dilatation part of the potential energy functions for different type of deformations, which is expressed by the function A_{ij}. By subtracting the experimental stress data, T_{ii}, from Aij, one obtains the contribution from the shear part of the potential function which is denoted as ΔT_{ii}. In the following, various deformations such as uniaxial tension, equal biaxial tension, and other biaxial tension are illustrated,

For various deformations, the contributions from the shear part of potential energy function ATii are:

(1) <u>Uniaxial tension</u> (T_{11}, $T_{22} = T_{33} = 0$, λ_1^*, $\lambda_2^* = \lambda_3^*$, $\lambda_1^* \lambda_2^* \lambda_3^* = 1$)

$$J\,T_{11} = A_{12} + \lambda_1^* h'(\lambda_1^*) - (\lambda_2^*) h'(\lambda_2^*) \tag{28}$$

or

$$\Delta T_{11}{}^{u} \equiv J\,T_{11} - A_{12} = \lambda_1^* h'(\lambda_1^*) - \lambda_2^* h'(\lambda_2^*) \tag{29}$$

where AT, $_1{}^{u}$ is the contribution from the shear part of potential energy function.

(2) Equal biaxial tension ($T_{11} = T_{22}, T_{33} = 0, \lambda_1^* = \lambda_2^* \lambda_1^* \lambda_2^* \lambda_3^* = 1$)

$$J\,T_{11} = A_{13} + \lambda_1^* h'(\lambda_1^*) - \lambda_3^* h'(\lambda_3^*) \tag{30}$$

or

$$\Delta T_{11}^{EB} \equiv J\,T_{11} - A_{13} = \lambda_1^* h'(\lambda_1^*) - \lambda_3^* h'(\lambda_3^*) \tag{31}$$

(3) Other biaxial tension ($\alpha = 5.7°, 26.5°$) ($T_{11}, T_{22}, T_{33} = 0, \lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_1^*, \lambda_2^* \lambda_3^* = 1$)

$$J\,T_{11} = A_{13} + \lambda_1^* h'(\lambda_1^*) - \lambda_3^* h'(\lambda_3^*) \tag{32}$$

$$J\,T_{22} = A_{23} + \lambda_2^* h'(\lambda_2^*) - \lambda_3^* h'(\lambda_3^*) \tag{33}$$

m

$$\Delta T_{11}^{B} \equiv J\,T_{11} - A_{13} = \lambda_1^* h'(\lambda_1^*) - \lambda_3^* h'(\lambda_3^*) \tag{34}$$

$$\Delta T_{22}^{B} \equiv J\,T_{22} - A_{23} = \lambda_2^* h'(\lambda_2^*) - \lambda_3^* h'(\lambda_3^*) \tag{35}$$

All ATii data for uniaxial, equal biaxial, and other biaxial deformations are presented in Table 1.

Using the data in Table 1, the function $h(\lambda_i^*)$ will be determined in terms of (a) the analytical type of representation, or (b) the numerical-graphical representation.

a. Analytical Type Representation

$$h(\lambda_i^*) = \sum_{n=1} \frac{\mu_n}{\alpha_n} \lambda_i^{*\alpha_n} \tag{36}$$

By nonlinear curve fitting to the data, one has the constants μ_n and α_n, where n = 3 is sufficient to describe the response. The various constants are shown in the following

Coefficients	
$\alpha 1$	1
$\alpha 2$	2
$\alpha 3$	3
$\mu 1$	121282.177623
$\mu 2$	4.460059922
$\mu 3$	-304.8057043

The comparison of calculated and measured data of ATii are shown from Figs. (1) to (6), respectively, for uniaxial, equal biaxial, and other biaxial deformations. The. prediction is reasonably good!

b. Numerical-Graphical Representation

$$H(\lambda_1^*, \lambda_2^*, \lambda_3^*) = h(\lambda_1^*) + h(\lambda_2^*) + h(\lambda_3^*) \tag{3-I}$$

and $$\lambda_1^* \lambda_2^* \lambda_3^* = 1 \tag{38}$$

(1) Uniaxial tension:

$$\Delta T_{II}^{u} = \lambda_1^* h'(\lambda_1^*) - \lambda_1^{*-1/2} h'(\lambda_1^{*-1/2}) \equiv F(\lambda_1^*) \tag{39}$$

(2) Equal biaxial tension:

$$\Delta T_{II}^{EB} = \lambda_1^* h'(\lambda_1^*) - \lambda_1^{*2} h'(\lambda_1^{*2}) \equiv G(\lambda_1^*) \tag{40}$$

From Eqs. (39) and (40), one observes that by replacing λ^* with $\lambda^{*1/4}$ in Eq. (40), and subtracting [he resulting equation from Eq. (39), one obtains

$$\lambda^* h'(\lambda^*) - \lambda^{*1/4} h'(\lambda^{*1/4}) = F(\lambda^*) - G(\lambda^{*1/4}) \tag{41}$$

By repeatedly replacing λ^* with $\lambda^{*1/4}$ and subtracting the previous equation, one obtains

$$\lambda^* h'(\lambda^*) - h'(1) = \sum_{m=0}^{m} \left\{ F\left[\lambda^{*(1/4)^m}\right] - G\left[\lambda^{*(1/4)^{(m+1)}}\right] \right\} \tag{42}$$

Thus one can obtain h' (λ^*) numerically from the uniaxial and equal biaxial tension data, using the recursion formula Eq. (42).

ATii" of the uniaxial data is shown in Fig. 7. ΔT_{II}^{EB} of the equal biaxial data is shown in Fig. 8. By using the recursion formula in Eq. (42), one obtains the graphical representation of h' (A*), which is shown in Fig. 9.

l-o check the validity of the graphical form h', the measured data and the calculated data are compared for uniaxial, equal biaxial, a = 26.5° and a = 5.7° response as shown from Figs. 10 to 15, respectively. The prediction is quite good.

CONCLUSIONS

It has been shown that, although the nonlinear response of the solid propellant is quite complex. it can be adequately described by the nonlinear theory of dilatable materials using a new set of volume-preserved invariant, Γ_i. The dilatation part and the shear part of the potential functions can be determined separately by this method, which provides a very powerful tool to attack the problem. If one follows the classical nonlinear theory of elasticity, using the first principal invariants, II, 12 and 13, it is easy to get lost in determining the potential function with these three independent variables. If one resorts to using brute force by expanding W in terms of II, 12, and 13 with many higher order terms, it would be difficult - if not impossible - to determine the W. Our method not only provides effective ways to solve the problem, but also provides a clear physical picture of the potential function in the role of overall stress-strain response.

In this paper, the shear part of the potential function in terms of the volume-preserved stretch ratios $\lambda_i^* = \frac{\lambda_i}{I_3^{1/6}}$ (i = 1, 2, 3) is determined. This effort to use two sets of variables (i.e., the volume-preserved invariant I_i^* and the volume-preserved principal stretch ratios) is, in part, due to the existence of two types of nonlinear finite-element computer codes, which are now available on the market and which were developed in terms of either principal invariant or principal stretch ratios.

REFERENCES

[1] Peng, S. T. J. "Constitutive Equations of Solid Propellants With Volume Dilatation Under Multiaxial Loading-Theory of Dilatation and Dewetting Criterion, " presented at the 1992 JANNAF Propulsion Meeting. Indianapolis, Indiana, February 24-27, 1992.

[2] Peng, S. T. J, "Constitutive Equations of Solid Propellants with Volume Dilatation Under Multiaxial Loadings - Determination of the Shear Part of the Potential Function," CPIA Publication 591, November 1992.

Table 1. Measured values of ΔT_{ij} for unaxial, equal biaxial, and biaxial angles of α = 26.5° and 5.7°

	Lam_1	lam_2	/am_;	lam_ i_	lam_2_•	lam_3_	del_T11_A measured	del_T11_A cal
Uni	1.06	0.971[	0.9711	1.05962	0.971457	0.971457	3705646	3906300424
	1.08	0.9637	0.963	1.07891	0.962733	0.962733	45.7412	47.46427265
	1.1	0.956:	0.956:	1.09?8?	0.954407	0.954407	5 1 17169	55.01562178
$\Delta T_{11}{}^{U}$	1.12	0.950(	0.950(	1,11552	0.946804	0946804	58 62526	61.46725604
	1.14	0.9459	0'.945!	1"13250	0.93968	0.93960	M 0 1 0 2 4	67.06799977
	1.16	0.942:	0.942:	1.14862	0.933062	0.933062	68.39539	71.85016828
	1.18	0.938{	0.938{	"1164671	0.92661	"0.92s%1	70.90113	76.07689282
	1.2	0.936:	0.936:	1.17989	0.920615	0.920615	70.95367 "	79.58767528
	Lam_ 1	lam_2	lam_3	'am_1_	'am_ 2_•	lam_3_'	del_T11_B measurec	del_T11_B cal
Equal Biaxia	1,01	1.01	0.9832	1 .00900!	1.009005	0.982231	12,06993601	10.90315779
	1,02	1.02	0.9679	1.01763	1,01763	0.965651	22.37922372"	28.67451644
$\Delta T_{11}{}^{EB}$	"-103	1,03-	0.9539	"102591!	1.025915	0.950117	33.56720466	38.21268918
	1,04	1.04-	0.9412	1,03383:	1.033833	0.935619	44.74680062	47.44030626
	1 :oi"-	1.05	0.9288	1.04173	1.041731	0.9?1486	54.78087049	56.72140976
	1.06	1.06	0.9185	1.04892	1.04892	0.908099	63.06720658	65.21983622
	-lo?-	1,07	"0.9093	1.05574!	1.055745	0.897186	71.57449874	73.31127171
	1.08	1.08	0.9019	1,06191:	'1.061912	0.680795	79.14339548 "	80.63973744
	1.09	1.09	0.8955	1.06771'	1.067711	0.877188	8623241114	87.52235307
	1.1	t.1	0.8901	1.07312;	1.073127	0,868355	94.68629834	93.94613208
	"1.11	-'1.11-"	0.8863	1.07780(	1.077906	0.860674	103.5311819	99.60965315
	1.12	1.12	0.8833	1.08235(	1.082356	0.853611	113.8672625	104.8659709
	1.13	1.13	0.881-"	1.08651	1.086511	0.847094	125.4907161	109.7647515
	1.14	1.14	0.8795	1,09032[	1.090326	0.841177	139.4787682	114.2429357
	1.15	1.15	0.8771	1.094501	1.094501	0.834771	154.388054	119.1357246
	lam_1	lam_2	lam_3	9m_ 1_	9—m_2_•	lam_3_•	lel_T11_C measured	del_T11_C cal
x=26.5	1.02	1.0025	0.979	1,019634	1.002141	0.978649	2245208	24.03989828
	1.03-	1.0045	).9685	1.029299	1.003817	0.967841	30.94567	. 31.64071523"
	-1.04'"	1.007	).9592	1.038427	1.005477	0.957749	30"67299	38.81212865
$\Delta T_{11}{}^{B}$	1.05	1.01	0.95	1.04? 397	1.007496	0.947645	45,63156 "	45.96129488
	1.06	1.014	).9411	1.055956	1.010132	0.93751	51 92293	53.03288734
	1.07	1.017	).9337	1.064338	1.011619	O 92876	5858224	59.4? 418024
	1.08	1.022"	).9259	1.072204	1.014623	0.919217	6421138	66.08174959
	-'1.09	1.026	).9192	1,08002	1.016606	0.910784	7019848	7219300626
	1.1	1.031	).9126	1.087465	1.019252	0.902201	75.650?	78" 29962915
	1."11	1.036	).9079	1.094165	1.02122	0.894948	82.06288	83.55021622
	1.12	1.04	0.904"	1.100892	.022257	0.888577,	87.68071	88.31622426
	1.13	1.045	"0.92	.099219	.016534	0.894939	1039013	8452680265"
	1.14	1.05	).8972	1.11321"	.025325	0.8;6116	9974816	9743441191
	1.15	1.056	).8943	.118792	.027343	0.8?0031	10G3485	1018060618
	-1.16-"	1.061	).8921	.124421	.028458	0.864738	112,0109	105.7069207
	1.17	1.066	0.89	.129991	.029547	0.859566	117.1259	109.5081816

Table I (continued)

	Lam_1	lam_2	lam_3	lam_1_'	am_2_'	lam_3._	del_T22D measured	dol. T22_D ca.
α=26.5	1.02-	1.0025	0.979	1.019634	1002141	0 97864	-1505775	1779953985
	1.03	1.0045	0.9685	1029299	1.003817	0 96784	2155478	?? 8251666
ΔT_{22}^{E}	1.04-	1.007	0.9592	1.038427	1.005477	0.95774'	2724i?4	2773412466
	1.05	1.0	0.95	1.047397	1.007496	0.94764	32"46603	3295675307
	1.06	1.014	0.9411	1.055956	1.010132	-0937-51	36.75906	'38 58068878
	1.07	1.017	0.9337	1.064338	1.011619	0.92876	40.37222	4329046282
	1.08	1.022	0.9259	1.072204	1.014623	0.91921	4 3 3 2 5 9 2	49055?9513'
	1.09	1.026	0.9192	1.08002	1.016606	0.91078	4 5 6 0 2 5 6	5403667092
	1.1	1.031	0.9126	1.087465	1.019252	0.90220	"4804121'	5943090976
	1.-1-	1.036	0.9079	1.094165	1.02122	0.89494	49.70917	6398372653
	1.12	-1.04	0.904	1.100892	1.022257	0.8885"7'	566453-	67.81747935
	1.13	1.045	0.9002	1,10722	1.023933	0.88205	5 2 1 3 8 1 1	720164861
	1.14	1.05	0.8972	1,11321	1.025325	0.87611(	54.37531	75 84600603
	1.15	1.056	0.8943	1.118792	1.027343	0.87003	5 8 . 0 4 6 6 3"	80.02429593
	1.16	1.061	0,8921	1.124421	1.026458	0.86473	61. 01143	8348931385
	1.17	1.066	0.89	1.129991	1.029547	0.85956	66.46781	8691625842
	Lam_1	lam_2	lam_3	am_1_.	am_2_	am._3_	del_T11_C measured	del T11 C cal
α=5.7	1.04	0.982	0.9792	1.039987	).981988	0.979188	2912121	30.37062155
	1.05 '	-o.9i9-	0.9748	1.049285	).978333	0.974136	3418681	35.12687017
ΔT_{11}^{B}	1.06	'0.977	0.9695	1.058579	0.97569	0.9682"	38.8937	40.13243535
	1.07	-0.976'	0.9638	1.067686	<973889	0.961716	43,27225	45.21853027
	1.08"""	0.975	0.9592	1.07641	).971759	0.956012	47,43904	49.79111333
	1.09	0.974	0.9551	1.084903	).969499	0.950687	52.4484	54.0604769
	1.1 "	-0.974"	0.9511	1.093117	).967906	094514!	56.81252	58.25591723
	1.1-1 '	0.974	0.9472	1.101239	).966313	0.939724	61.42745	62.31519781
	1.12	0.9745	0.9437	1.109022	).964948	0.93445	65.90205	66.16043095
	1.13	0.975	0.9412	1.110409	).963274	0".92988	70.22643	69.53218637
	iji4	0.9755	0.9388	1.12375	),961595	0.925418	74,0082	72.76839819
	1.15	0.9765	0.9359	1.131092	).960445	0 920512	7753790	76.15274727
	1.16	0.9775	0.9339	1.138063	).959014	0.916239	80.44589	79.10253735-
	1.17 --	0.978	0.9325	1.144971	).957079	0.912552	82.12308	81.67887291
	Lam.>	lam_2	lam_3	am_1_"	am_2_"	am_3_	del_T22D measured	9el_T22-.Dcal
α=5.7	1.04	0.982	3.9792	1,039987	),981988	).979188	12.77035	9.704557212
	1.05	0.979	).9748	1.049285	).978333	).974136	1 6 . 0 9 6 9	10,22332447
ΔT_{22}^{B}	1.06	0.977	).9695	1.058579	0.97569	0.9682	19. 34982	11.56015788
	1.07	0.976	).9638	1.067686	) 973869	).961716	22.03454 -	13.53544041
	1.08	0.975	).9592	1.07641	).971759	).956012	24.53934	T5.1OO98715
	1.09	0.974	).9551	1.084963	).969499	) 950687	2 5 . 9 2 2 9 8	16.48519239"
	1-.1-	0.974	).951 i	1.093117	).967906	).945149	2 9 0 4 4 8	18.30028928
	1.11	0.974	).9472	1.101239	).966313	1.939724	"3114097	20.11293853
	1.12	0.9745	),9437	1.109022	).964948	0.93445	32.61831 ' - "	21.99619632
	1.13	0.975	).9412	1.118-R	).963274	0.92980	3394045	"23.44864636
	1.14	0.9755	).9388	11 2375	).961595	) 925418	3540875	24.87566187
	1.15	0.9765	).9359	1.131092	).960445	).920512	364127	2679286452
	1.16	0.9775	).9339	1.138063	).959014	).916239	3765085	2829783693
	1.17	0.978	).9325	1.144971	957079	),912552	3903038	293042388

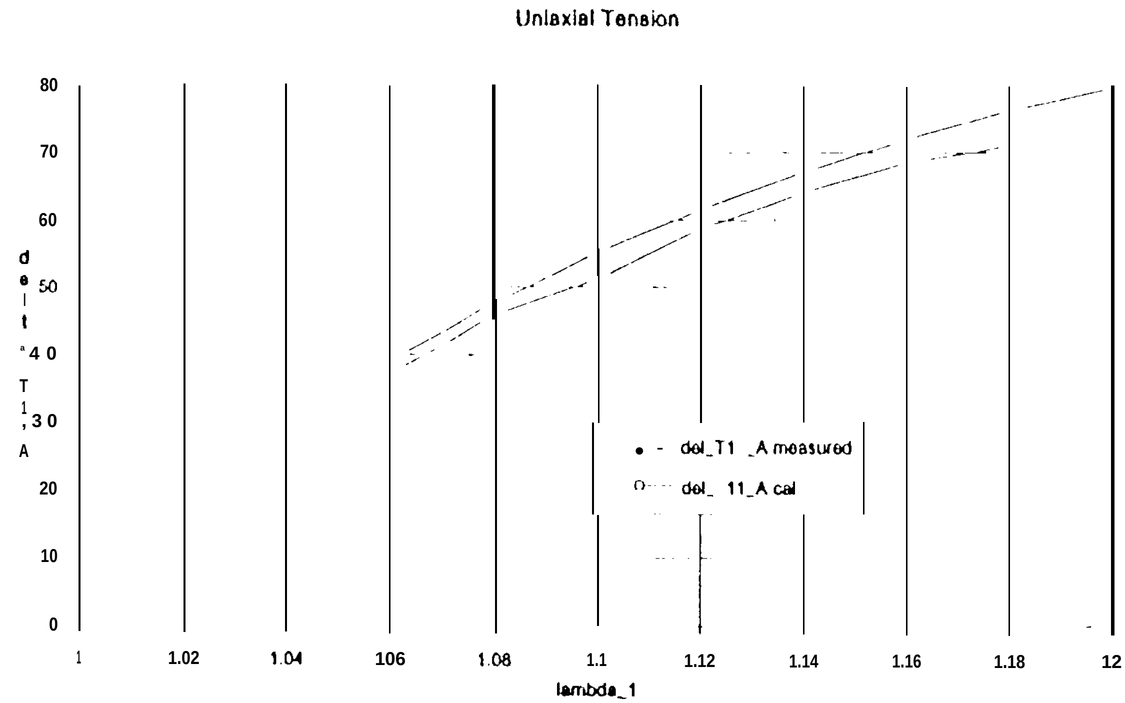


Fig. 1. Comparison of the calculated and measured data of ΔT_{11} for unaxial tension (RSRM, T = 25°C)

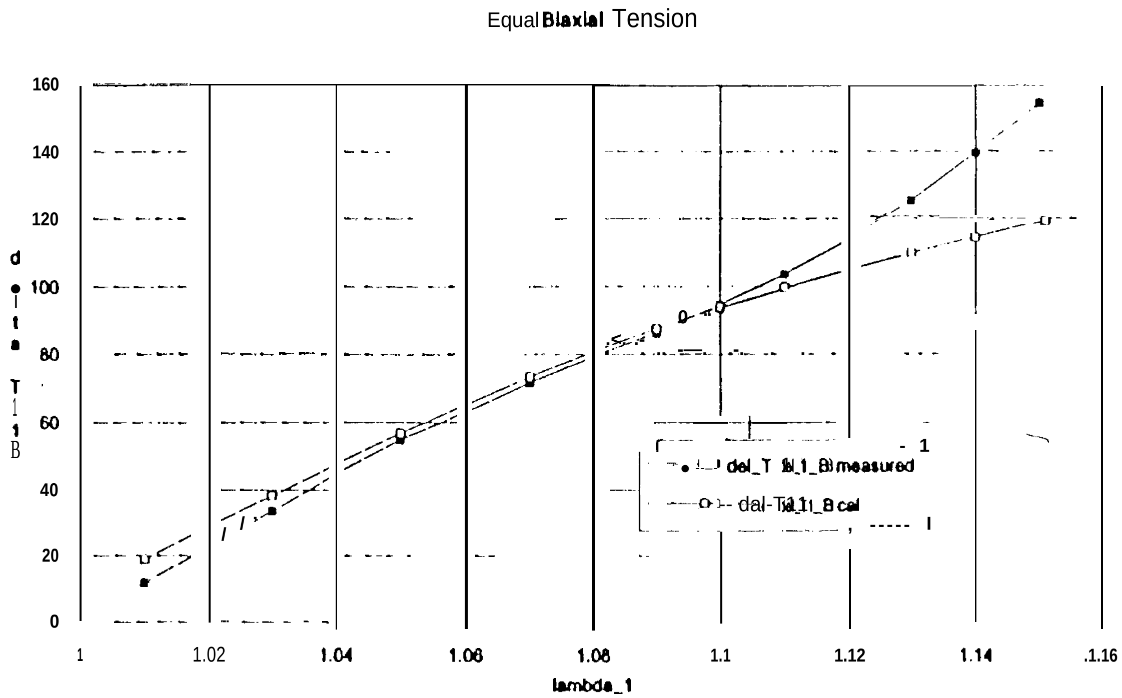


Fig. 2 Comparison of the calculated and measured data of ΔT_1 for equal biaxial tension (RSRM, T - 25°C)

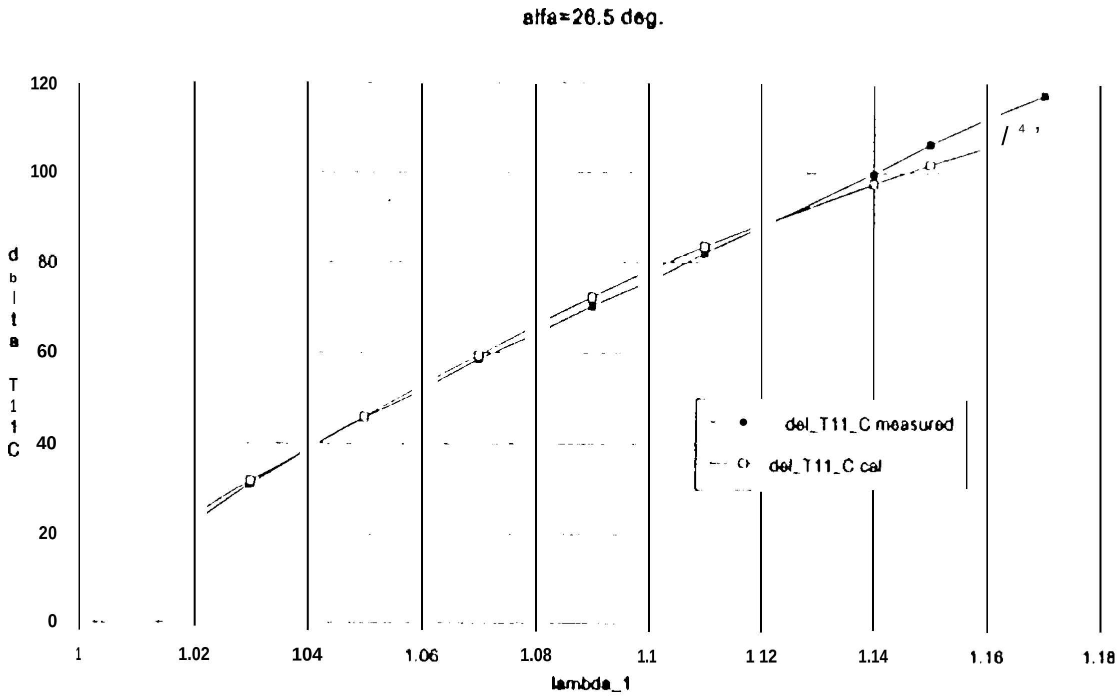


Fig. 3. Comparison of the calculated and measured data of ΔT_{11} for a = 26.5° (RSRM, T - 25°C)

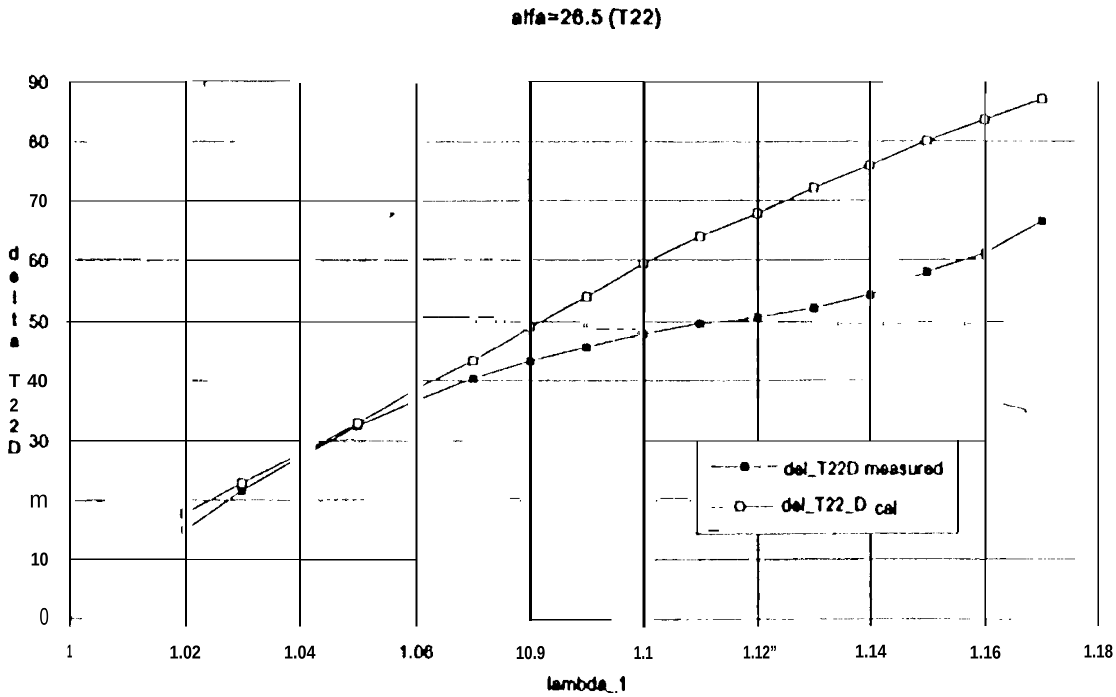


Fig. 4. Comparison of the calculated and measured data of ΔT_{22} for a -26.5" (RSRM, T - 25°C)

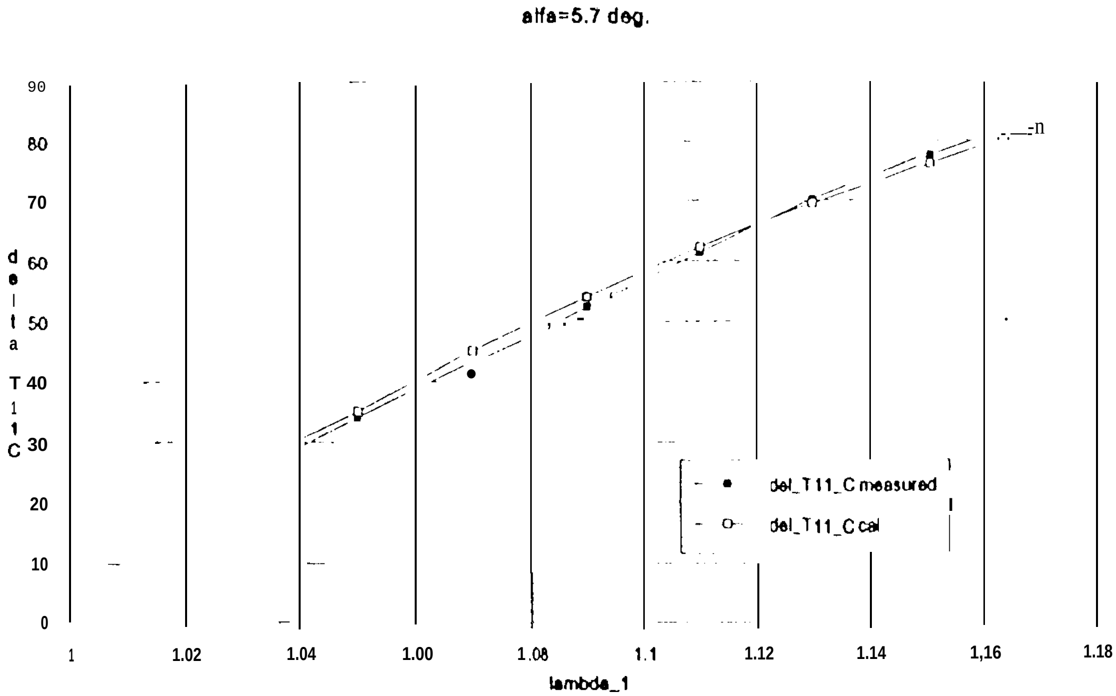


Fig. 5. Comparison of the calculated and measured data of ΔT_{11} for a -5.7° (RSRM, T = 25°C)

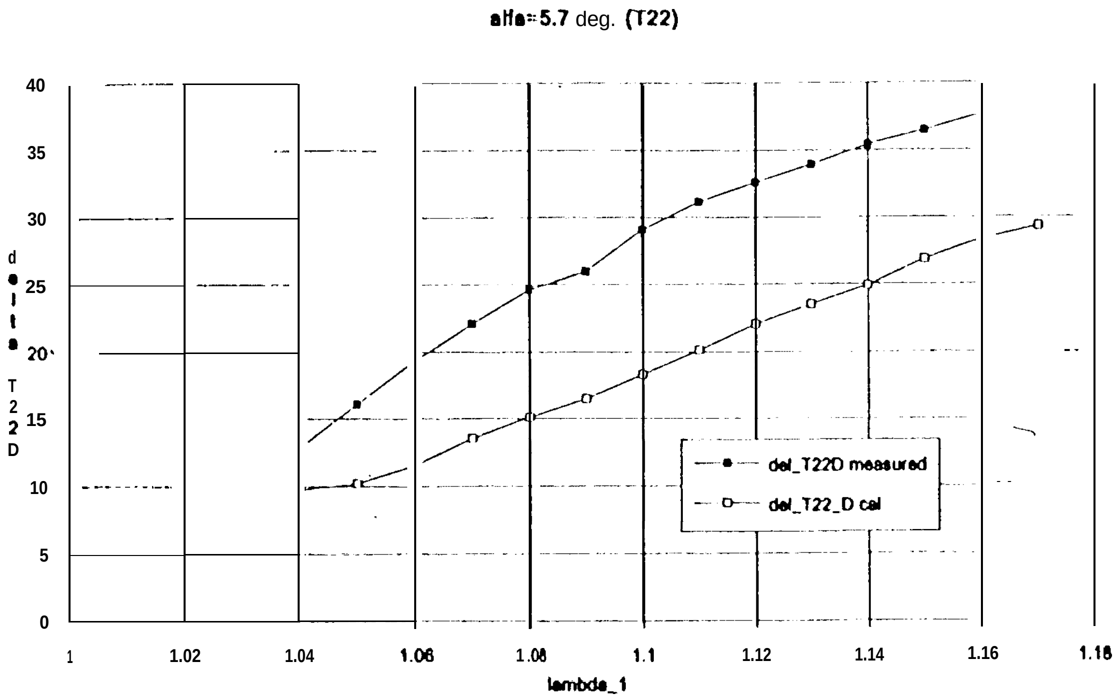
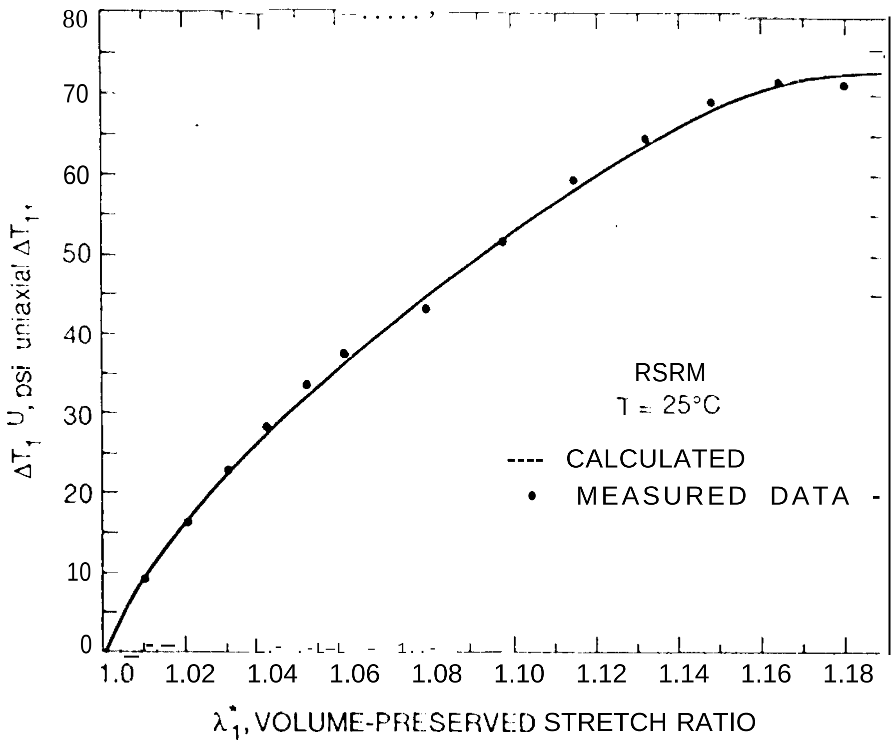


Fig. 6. Comparison of the calculated and measured data of AT22 for a -5.7° (RSRM, T - 25°C)



Fig, 7. Experimental data of T_{11}^{U} versus λ_1^* of uniaxial tension

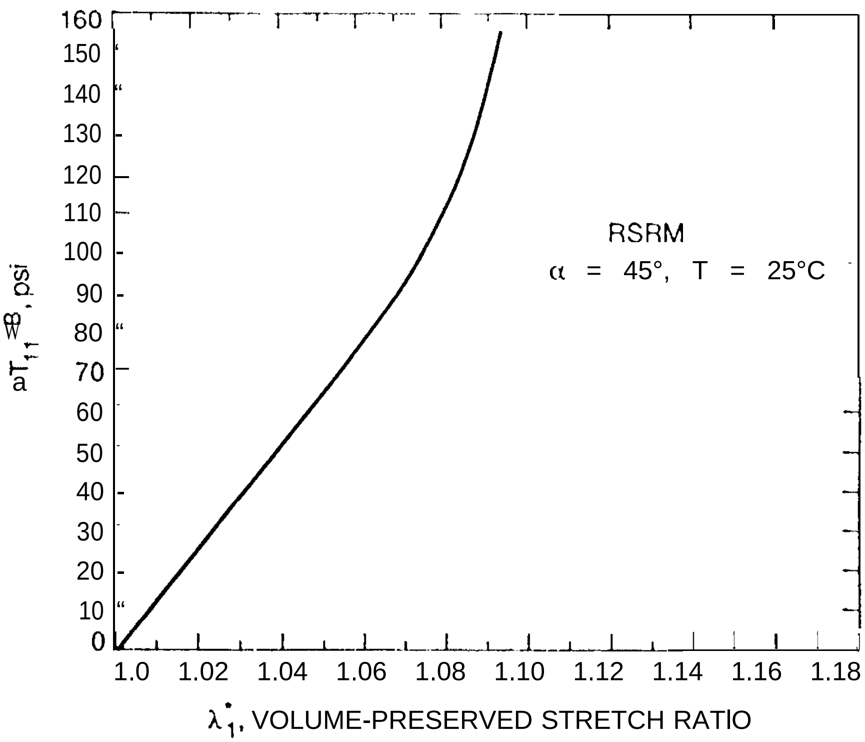


Fig. 8. Experimental data of ΔT_{11}^{EB} versus λ_1^* of equal biaxial deformation

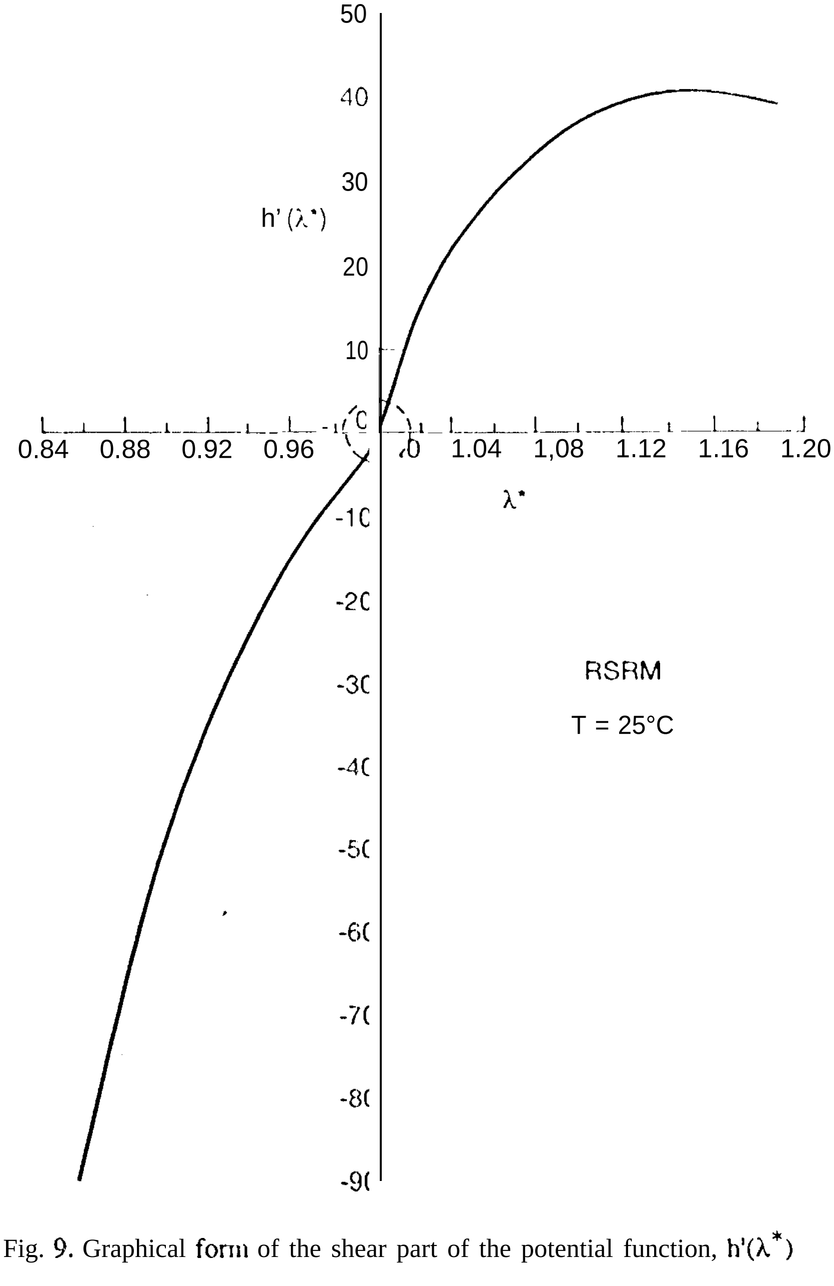


Fig. 9. Graphical form of the shear part of the potential function, $h'(\lambda^*)$

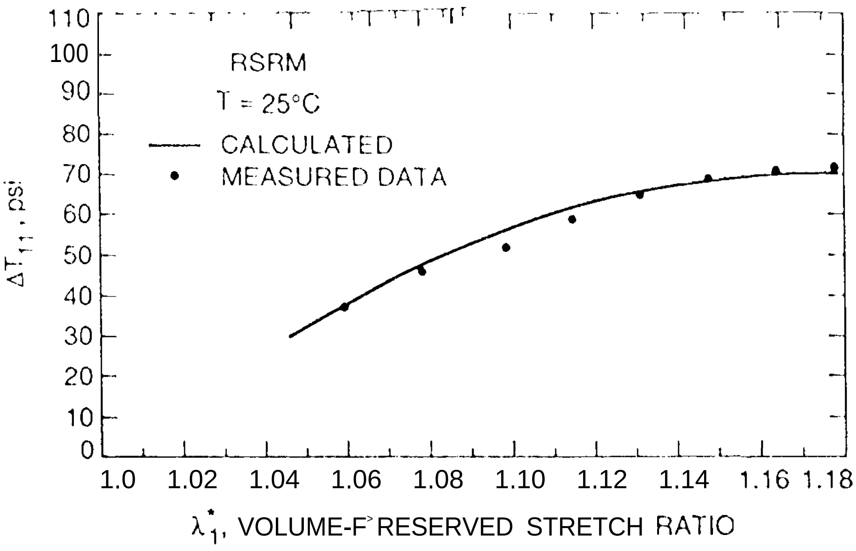


Fig. 10. Comparison of the calculated and measured data of ΔT_{11} for unaxial tension

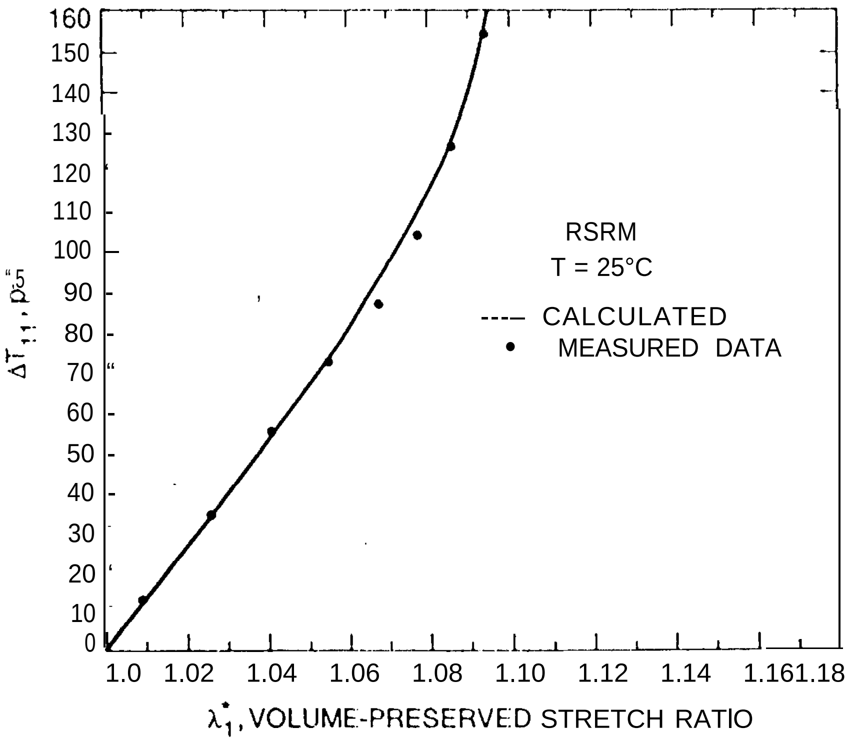


Fig. 11. Comparison of the calculated and measured data of ΔT_{11} for equal biaxial tension

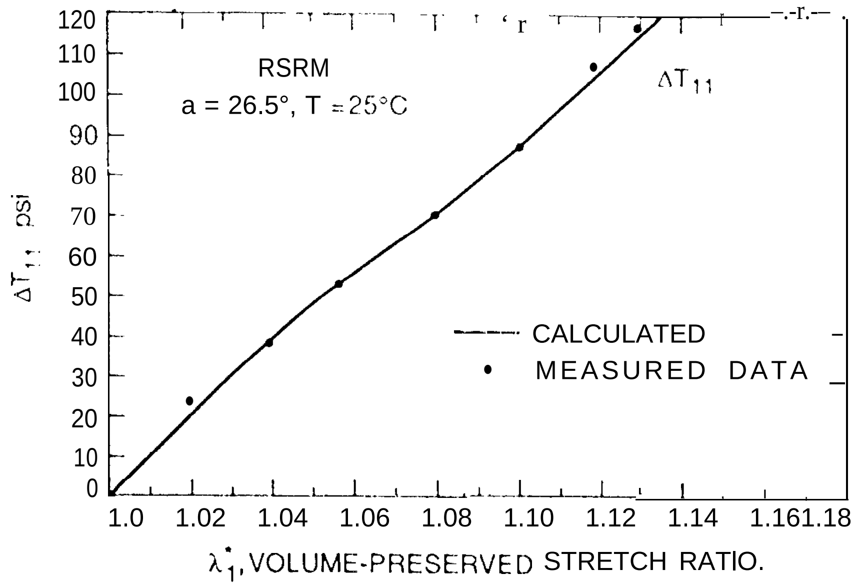


Fig. 12. Comparison of the calculated and experimental data of ΔT_{11} versus λ_1^* for a = 26.5°

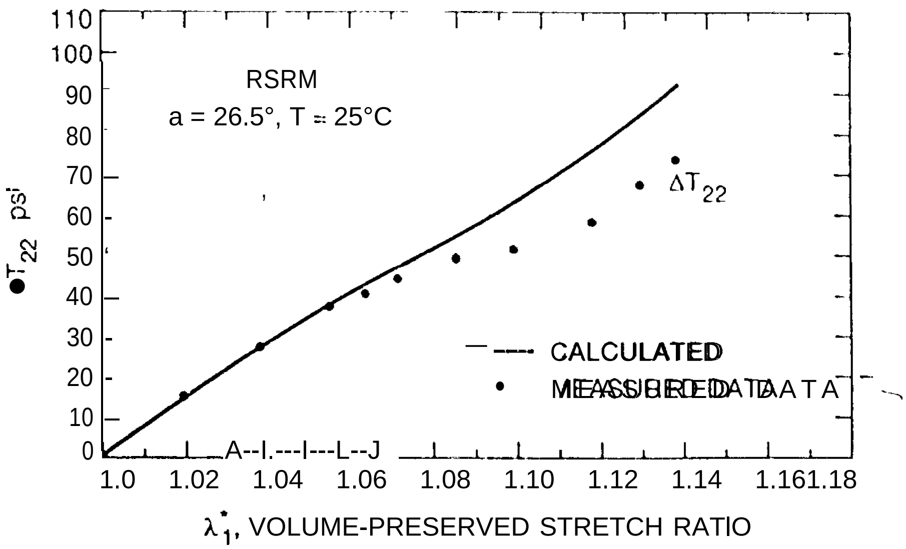


Fig. 13. Comparison of the calculated and measured data of ΔT_{22} λ_1^* for a -.26.5°

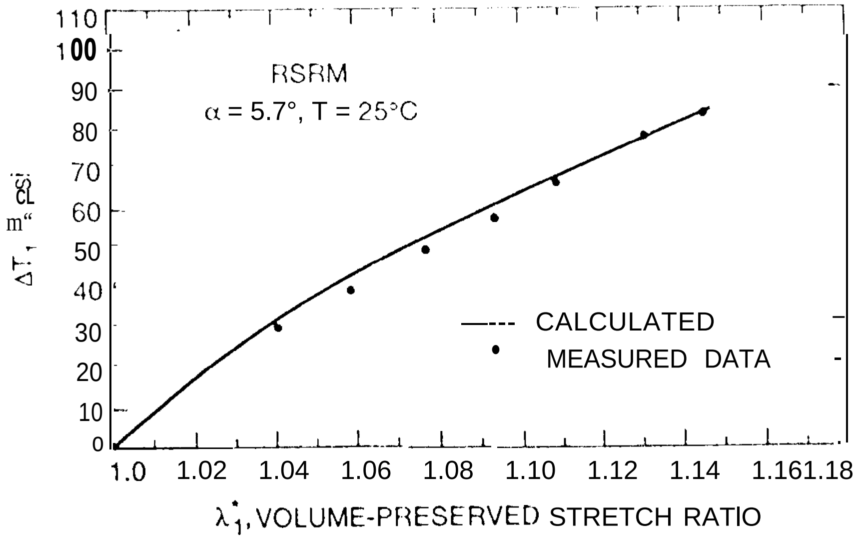


Fig. 14. Comparison of the calculated and experimental data of $\Delta T_{11}{}^{B}$ versus λ_1^* for $\alpha = 5.7°$

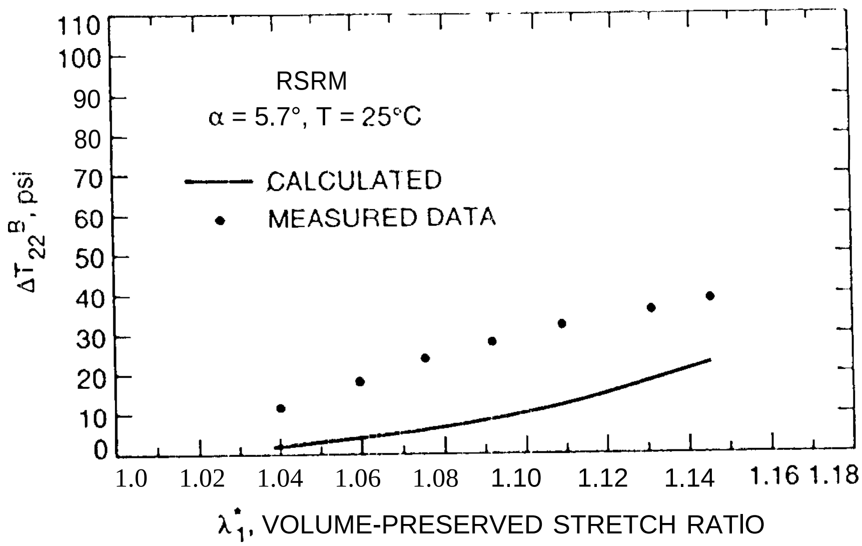


Fig. 15. Comparison of the calculated and measured data of $\Delta T_{22}{}^{B}$ versus λ_1^* for a -5.7°